

In[1]:= $\$Assumptions = \alpha \in \text{Reals} \ \&\& \ m \in \text{Integers} \ \&\& \ \alpha > 0 \ \&\& \ m \geq 1$

Out[1]= $\alpha \in \text{Reals} \ \&\& \ m \in \text{Integers} \ \&\& \ \alpha > 0 \ \&\& \ m \geq 1$

Volumen und Oberflaeche

In[2]:= $\text{Volumen}[C_ , R_ , N_] := C R^N$

In[3]:= $\text{Oberflaeche}[C_ , R_ , N_] := N C R^{N-1}$

Koeffizient C_{gerade}

In[4]:= $\text{Igerade} = \frac{1}{2} \int_0^\infty u^{m-1} E^{-\alpha u} du$

Out[4]= $\frac{1}{2} \alpha^{-m} \text{Gamma}[m]$

In[5]:= $\text{Solve}\left[2 m C_{\text{gerade}} \text{Igerade} = \left(\frac{\text{Pi}}{\alpha}\right)^m, C_{\text{gerade}}\right] // \text{FullSimplify} // \text{TraditionalForm}$

Out[5]/TraditionalForm=

$$\left\{\left\{C_{\text{gerade}} \rightarrow \frac{\pi^m}{m!}\right\}\right\}$$

Volumen und Oberflaeche fuer gerade N

In[6]:= $\text{Vgerade}[R_ , N_] = \text{Volumen}\left[\frac{\pi^{N/2}}{(N/2)!}, R, N\right] // \text{TraditionalForm}$

Out[6]/TraditionalForm=

$$\frac{\pi^{N/2} R^N}{\frac{N}{2}!}$$

In[7]:= $\text{Ogerade}[R_ , N_] = \text{Oberflaeche}\left[\frac{\pi^{N/2}}{(N/2)!}, R, N\right] // \text{TraditionalForm}$

Out[7]/TraditionalForm=

$$\frac{N \pi^{N/2} R^{N-1}}{\frac{N}{2}!}$$

■ Einheitskugel

In[8]:= $\text{Vgerade}[1, N]$

Out[8]/TraditionalForm=

$$\frac{\pi^{N/2}}{\frac{N}{2}!}$$

In[9]:= **Ogerade**[1, N]

Out[9]//TraditionalForm=

$$\frac{N \pi^{N/2}}{\frac{N}{2}!}$$

Koeffizient C_{ungerade}

In[10]:= **Iungerade** = $\int_0^{\infty} r^{2m} E^{-\alpha r^2} dr$

Out[10]= $\frac{1}{2} \alpha^{-\frac{1}{2}-m} \text{Gamma}\left[\frac{1}{2} + m\right]$

In[11]:= **Solve**[(2 m + 1) **Cungerade** **Iungerade** == $\left(\sqrt{\frac{\text{Pi}}{\alpha}}\right)^{2m+1}$, **Cungerade**] // **FullSimplify** // **TraditionalForm**

Out[11]//TraditionalForm=

$$\left\{\left\{\text{Cungerade} \rightarrow \frac{\pi^{m+\frac{1}{2}}}{\Gamma\left(m + \frac{3}{2}\right)}\right\}\right\}$$

Volumen und Oberflaeche fuer ungerade N

In[12]:= **Vungerade**[R_, N_] =

$$\text{Volumen}\left[\frac{\pi^{m+\frac{1}{2}}}{\text{Gamma}\left[m + \frac{3}{2}\right]} /. m \rightarrow \frac{1}{2} (N-1), R, N\right] // \text{FullSimplify} // \text{TraditionalForm}$$

Out[12]//TraditionalForm=

$$\frac{\pi^{N/2} R^N}{\Gamma\left(\frac{N}{2} + 1\right)}$$

In[13]:= **Oungerade**[R_, N_] =

$$\text{Oberflaeche}\left[\frac{\pi^{m+\frac{1}{2}}}{\text{Gamma}\left[m + \frac{3}{2}\right]} /. m \rightarrow \frac{1}{2} (N-1), R, N\right] // \text{FullSimplify} // \text{TraditionalForm}$$

Out[13]//TraditionalForm=

$$\frac{2 \pi^{N/2} R^{N-1}}{\Gamma\left(\frac{N}{2}\right)}$$

■ Einheitskugel

In[14]:= **Vungerade**[1, N]

Out[14]//TraditionalForm=

$$\frac{\pi^{N/2}}{\Gamma\left(\frac{N}{2} + 1\right)}$$

In[15]:= **Oungerade**[1, N]

Out[15]//TraditionalForm=

$$\frac{2\pi^{N/2}}{\Gamma\left(\frac{N}{2}\right)}$$

Ueberpruefung fuer $N = 2$ und $N = 3$

In[16]:= **Vgerade**[R, 2]

Out[16]//TraditionalForm=

$$\pi R^2$$

In[17]:= **Ogerade**[R, 2]

Out[17]//TraditionalForm=

$$2\pi R$$

In[18]:= **Oungerade**[R, 3]

Out[18]//TraditionalForm=

$$4\pi R^2$$

In[19]:= **Vungerade**[R, 3]

Out[19]//TraditionalForm=

$$\frac{4\pi R^3}{3}$$